# Assignment #1

# Mujtaba Khan

# 250966314

Problem 1:

**(p→q) ⇔ (¬q→¬p)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **p→q** | **¬q→¬p** | **(p→q) ⇔ (¬q→¬p)** |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

1. **(p**∧**q) ⇔ (¬q**∧**¬p)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **p** | **q** | **p∧q** | **¬q∧¬p** | **(p∧q) ⇔ (¬q∧¬p)** |
| T | T | T | F | F |
| T | F | F | F | T |
| F | T | F | F | T |
| F | F | F | T | F |

Therefore, since the truth values of the new tautology do not match the original tautology the new one is to be considered **False**.

**(p↔q)⇔((p→q)∧(q→p))**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **p↔q** | **p→q** | **q→p** | **(p→q)∧(q→p)** | **(p↔q)⇔((p→q)∧(q→p))** |
| T | T | T | T | T | T | T |
| T | F | F | F | F | F | T |
| F | T | F | T | T | F | T |
| F | F | T | T | T | T | T |

1. **(p↔q)⇔((p∨q)∧(q∨p))**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **p↔q** | **p∨q** | **q∨p** | **(p∨q)∧(q∨p)** | **(p↔q)⇔((p∨q)∧(q∨p))** |
| T | T | T | T | T | T | T |
| T | F | F | T | T | T | F |
| F | T | F | T | T | T | F |
| F | F | T | F | F | F | F |

Therefore, since the truth values of the new tautology do not match the original tautology the new one is to be considered **False**.

Problem 2:

1. For any two even integers, there exists a third integer (even or odd) the double of which is equal to the sum of the first two integers.

**Statement Translation**: ∃z ∀x ∀y (f(z) ⇔ P(x,y))

**Assumptions**: x, y are even numbers, z is an even or odd number, f(z) multiplies z by 2 making it into an even number, P(x,y) adds x and y.

**Proof**: Assume that since x, y and z will always be even numbers we can quantify them to; 2a, 2b, 2c, respectively. So now we sub the values into the predicate statement.

f(2c) = P(2a,2b)

2(2c) = 2a + 2b

4c = 2a + 2b

2c = a + b

**Example**: x = 2, y = 4. x + y = 6. 6/2 = 3. c = 3.

Therefore, since this equation does hold it shows that the addition of 2 numbers can always be made up by the double of a different number.

2. For any two odd integers, there exists a third integer (even or odd) the triple of which is equal to the sum of the first two integers.

**Statement Translation**: ∃z ∀x ∀y (f(z) ⇔ P(x,y))

**Assumptions**: x, y are odd numbers, z is an even or odd number, f(z) multiplies z by 3, P(x,y) adds x and y.

**Proof by Contradiction**: Assume that x, y are odd and that the third integer z is even and does not equal the sum of x and y. Since x and y are odd we can quantify them as 2a + 1, 2b + 1, respectively. Since we are assuming z is even it can be quantified as 2c. Now we sub the values into the predicate statement.

f(2c) = P((2a+1), (2b+1)) 3c = a + b + 1 c = 2/3

3(2c) = 2a + 2b + 2 3c = a + b

6c = 2a + 2b + 2 3c = 1 + 1

3c = a + b + 1 3c = 2

**Example**: x = 1, y = 1. x + y = 2. There is no 3rd **integer** which can triple to 2.

Therefore, since this equation requires a non-integer value it contradicts the elementary statement and does not hold, it reaffirms the previous statement wherein we assumed the 2 statements are not equivalent. This then proves that when x, y are odd numbers and z is any number there are instances where the addition of x, y is not equal to the tripling of any integer.

Problem 3:

**Terminology**: CPT = Coconut Palm Tree, SB = Shipwreck on Beach, FI = Far from Island, NC = Near Cave, C = Cave

**Order Solved**: 5 🡪 2 🡪 4 🡪 1 🡪 3

**Solution**: The Treasure is hidden in the **Cave**

**Process**:

5. There is no coconut palm tree near the cave

This is the given value from which we start our analysis

2. There is a coconut palm tree growing either at the far end of the island or near the cave

CPT 🡪 FI \/ NC True \/ False 🡪 True, there is a CPT at the far side of the island

4. If there is a coconut palm tree at the far end of the island, then there is no shipwreck on the beach

CPT FI 🡪 ¬SB True 🡪 False 🡪 True, there is no shipwreck on the beach

1. If there is an old shipwreck near the beach, then the treasure is buried under a coconut palm tree

SB 🡪 CPT False 🡪 False 🡪 False, there is no treasure under the CPT

3. Either there is a shipwreck near the beach, or the treasure is hidden in a cave

SB \/ C False \/ True 🡪 True, the treasure is hidden in a cave

Problem 4:

**Terminology**: MS = Multiuser State, ON = Operating Normally, KF = Kernel Functioning, IM = Interrupt Mode

**Order** **Solved**: 5 🡪 4 🡪 1 🡪 2 🡪 3

**Solution**: **True**, the following set of propositions are consistent since all the values are True

**Propositions**:

1. The system is in multiuser state if and only if it is operating normally

MS ⇔ ON False ⇔ False 🡪 True The system is not operating normally

2. If the system is operating normally, the kernel is functioning

ON 🡪 KF False 🡪 False 🡪 True The system Kernel is not functioning

3. The kernel is not functioning, or the system is in interrupt mode

¬KF \/ IM False \/ True 🡪 True The system is in IM, the K is not F

4. If the system is not in multiuser state, then it is in interrupt mode

¬MS 🡪 IM True 🡪 True 🡪 True The system is in IM not MS

5. The system is in interrupt mode.

IM True 🡪 True The system is in IM

Problem 5:

**p ∧ (q ∨ ¬p) ∧ (¬q ∨ ¬r)**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **¬p** | **¬q** | **¬r** | | **(q ∨ ¬p)** | **(¬q ∨ ¬r)** | **p ∧ (q ∨ ¬p) ∧ (¬q ∨ ¬r)** |
| T | T | T | F | F | | F | T | F | F |
| T | T | F | F | F | | T | T | T | T |
| T | F | T | F | T | | F | F | T | F |
| T | F | F | F | T | | T | F | T | F |
| F | T | T | T | F | | F | T | F | F |
| F | T | F | T | F | | T | T | T | F |
| F | F | T | T | T | | F | T | T | F |
| F | F | F | T | T | | T | T | T | F |

Therefore, Statement 1 **is** **Satisfiable**

**p ∧ (q ∨ ¬p) ∧ (¬q ∨ ¬p)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **¬p** | **¬q** | **(q ∨ ¬p)** | **(¬q ∨ ¬p)** | **p ∧ (q ∨ ¬p) ∧ (¬q ∨ ¬p)** |
| T | T | F | F | T | F | F |
| T | F | F | T | F | T | F |
| F | T | T | F | T | T | F |
| F | F | T | T | T | T | F |

Therefore, Statement 2 **is not Satisfiable**